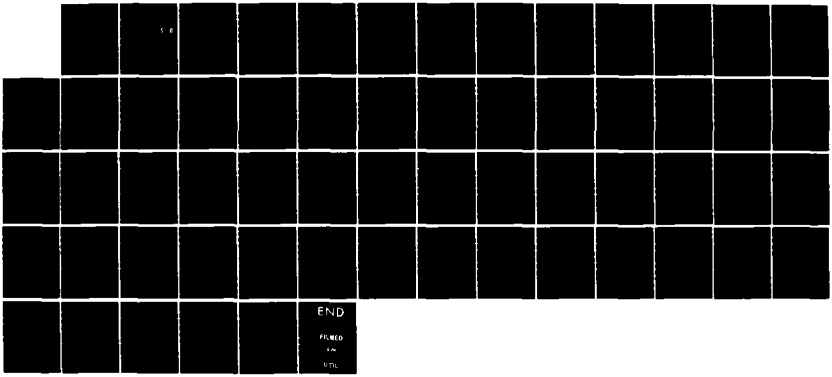
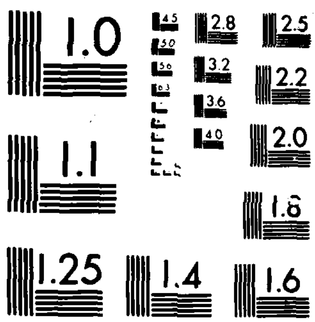
AD-A163 993 ROBUST ESTIMATION OF THE LOCATION PARAMETER OF LIFE DISTRIBUTIONS(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGINEERING 1/1

UNCLASSIFIED L M ALLEN DEC 85 AFIT/GSO/MA/85D-1 F/G 12/1 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

ROBUST ESTIMATION OF THE LOCATION
PARAMETER OF LIFE DISTRIBUTIONS

THESIS

Linda M. Allen
Captain, USAF

AFIT/GSO/MA/85D-1

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

1

AFIT/GSO/MA/85

ROBUST ESTIMATION OF THE LOCATION PARAMETER OF LIFE DISTRIBUTIONS

THESIS

Linda M. Allen
Captain, USAF

AFIT/GSO/MA/85D-1

AFIT/GSO/MA/85

ROBUST ESTIMATION OF THE LOCATION PARAMETER OF LIFE DISTRIBUTIONS

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Space Operations

Linda M. Allen, B.S.

Captain, USAF

December 1985

Approved for public release; distribution unlimited

Preface

The purpose of this study was to investigate the feasibility of using the two-parameter negative exponential distribution, with scale set equal to one, as a baseline for a robust estimator for the location parameter (minimum life) of selected life distributions. This estimator could be used in determining cost and performance estimates for systems containing many electronic components (e.g., satellites), where expense and small quantities make it impractical to conduct enough tests to determine underlying time to failure distributions with certainty.

A computer model was built using Monte Carlo techniques to generate time to failure data for several underlying distributions. Five estimates for the minimum life were computed from this data and compared to the actual minimum life. The best estimator of the five being compared was shown to be the minimum variance, unbiased, maximum likelihood estimator for the minimum life from the two-parameter negative exponential distribution. However, the limits of the usefulness of this estimator still need to be determined.

The development of this thesis required a great deal of help from others. I would like to thank my thesis advisor, Dr. Albert H. Moore, for his guidance and

technical assistance. My limited theoretical knowledge could have made this thesis an impossible task. His patient explanations of underlying theory and its relationship to the real world quite often lifted a very dense fog. I would also like to thank my reader, Dr. Joseph T. Cain. His feedback concerning the written portion of my thesis was invaluable.

Finally, I would like to thank my fellow classmates, the finest group of people I have ever worked with. Their support and camaraderie over the last 18 months helped ease the way through a difficult program. In particular I would like to thank: Major Dennis Charek and Captains Karen Barland, Jim Porter, and John Sours for their help in finding background information and debugging my computer program; Captains Jim Martin and Norm Jarvis for their moral support at my thesis defense - the morning after Thanksgiving; and First Lieutenant Stacy Brodzik for her support and encouragement, and for letting me know when I was taking things too seriously.

Linda M. Allen

Table of Contents

		Page
Preface		ii
List of Figures		vi
List of Tables		vii
Abstract		viii
I.	Introduction	1
	Problem	2
	Objectives	2
	Scope	2
II.	Background	4
	Life Distributions	4
	The Exponential Distribution	6
	The Weibull Distribution	7
	The Gamma Distribution	8
	The Log-normal Distribution	9
	Robust Estimation	11
	Minimum Distance Estimation	12
	The Kolmogorov Statistics	12
	The Cramer-von Mises Statistic	13
	The Anderson-Darling Statistic	13
	Measures of Effectiveness	13
	Mean Square Error	13
	Relative Efficiency	14
III.	Methodology	15
	Overview of Method	15
	Random Number Generation	15
	Parameter Estimation	16
	Maximum Likelihood Estimation	16
	Distance Estimation	16
	Investigation of Robustness of Estimator	16
IV.	Results	18
V.	Conclusions and Recommendations	20
Appendix A:	Graphs of Representative Probability Density Functions	21
Appendix B:	Tables I-IV, Mean Square Errors for Location Parameter Estimates	26

Appendix C: Tables V-VIII, Relative Efficiencies of Location Parameter Estimates 31

Appendix D: Sample Computer Program 36

Bibliography 45

Vita 46

List of Figures

Figure | Page
1. PDFs for several exponential distributions 22
2. PDFs for several gamma distributions when $\theta=1$ .. 23
3. Weibull PDFs for $\nu=0$, $\alpha=1$, $\beta=\frac{1}{2}$, 1, 2, 4 24
4. Lognormal density function and illustrations ... 25

List of Tables

Table		Page
I.	Mean Square Errors for Location Parameter Estimates (Sample size=8)	27
II.	Mean Square Errors for Location Parameter Estimates (Sample size=12)	28
III.	Mean Square Errors for Location Parameter Estimates (Sample size=16)	29
IV.	Mean Square Errors for Location Parameter Estimates (Sample size=20)	30
V.	Relative Efficiencies of Location Parameter Estimates (Sample size=8)	32
VI.	Relative Efficiencies of Location Parameter Estimates (Sample size=12)	33
VII.	Relative Efficiencies of Location Parameter Estimates (Sample size=16)	34
VIII.	Relative Efficiencies of Location Parameter Estimates (Sample size=20)	35

Abstract

This investigation determined that for time to failure distributions that are moderate deviations from the negative exponential distribution, a robust estimate of the minimum life could be arrived at by assuming the underlying distribution was exponential and using the minimum variance, unbiased, maximum likelihood estimator. It was found that estimators using the minimum distance statistics of Kolmogorov, Cramer-von Mises, and Anderson-Darling did not perform well with the asymmetric distributions explored in this thesis. However, they may still prove useful for life distributions with larger shape parameters (i.e., for distributions that are not "moderate" deviations from the negative exponential).

The analysis was accomplished by using Monte Carlo techniques to generate random samples of time to failure data from specific distributions, and using this empirical data to estimate the actual minimum life of the distribution. Five estimators were explored: the minimum variance, unbiased, maximum likelihood estimator of the two-parameter negative exponential distribution; the first ordered statistic; and the three minimum distance methods (the theoretical distribution was assumed negative exponential). The performance of these estimators was evaluated by comparing their mean square

errors with the mean square error of the chosen "best" estimator.

Robust Estimation of the Location Parameter of Life Distributions

I. Introduction

Space assets have been playing an ever greater role in the defense of this nation. Satellites can be placed at altitudes that give them a world view. Unfortunately, once in place these satellites are mostly inaccessible to us - if they break, they are expensive to repair, if they can be reached to repair at all. In addition, the space environment is hostile, which tends to increase the failure rate.

A typical satellite consists of hundreds of parts, all of which have to be made to exacting standards. There is a great deal of expense involved in the designing, building, testing, and placement of these satellites. In order to make decisions regarding the number of satellites to be built, or whether they should be built at all, there must be some way of estimating the expected minimum life of the satellite.

Testing different satellite components can produce a rough time to failure data base that can be used to estimate minimum life. In the case of a satellite, a very expensive piece of equipment, it is hoped that the expected minimum life is greater than zero.

Satellites are generally expensive and manufactured in small quantities. It may be difficult to collect enough time to failure data to identify the underlying distribution with certainty. What is desirable is an estimator which gives a useable value in most cases even when the assumption about the underlying distribution is incorrect. Such an estimator is termed "robust" (Parr and Schucany, 1979).

PROBLEM

A robust estimation method is desired for the location parameter of life distributions. This location parameter is the guaranteed minimum life.

OBJECTIVES

The overall objective of this thesis was to develop a robust estimator for the location parameter of life distributions, using the two-parameter negative exponential distribution, with scale equal to one, as the baseline.

SCOPE

Only four families of distributions were considered: exponential, Weibull, gamma, and log-normal. The sample sizes from each distribution were small. Four sample sizes were used: 8, 12, 16, and 20.

Three minimum distance methods were investigated: Kolmogorov, Cramer-von Mises, and Anderson-Darling. The robustness of the estimators will be evaluated using two measures of effectiveness: mean square error and relative efficiency.

II. Background

If the underlying distribution of a specific population is known, predictions can be made regarding samples from the population. For example, if it is known that the time between failures of the components of a particular type of satellite is exponentially distributed then predictions can be made concerning the performance of that satellite over a specified period of time.

Unfortunately, the underlying distribution of a population is usually something that has to be determined from a relatively small sample from that distribution. There is a great deal of literature concerning the determination of underlying distributions and the statistical inferences that may be made from these determinations. Miller's and Freund's text on probability and statistics, and Kapur's and Lamberson's text on reliability engineering were used extensively in the development of this thesis.

Life Distributions

This thesis is primarily concerned with life distributions. When electronic components are tested two of the more important statistics collected are time to failure and time between failures.

Time to failure has been modeled with several distributions, the exponential, gamma, Weibull, and

log-normal being of particular interest to this thesis (Banks and Carson, 1984:134). The exponential distribution is most commonly used for life testing applications, with the Weibull distribution probably the second most common (Kapur and Lamberson, 1977:233, 291).

Banks and Carson had the following to say about time to failure:

> "If only random failures occur, the time to failure distribution may be modeled as exponential. The gamma distribution arises from modeling standby redundancy where each component has an exponential time to failure... When there are a number of components in a system and failure is due to the most serious of a large number of defects, or possible defects, the Weibull distribution seems to do particularly well as a model... The log-normal distribution has been found to be applicable in describing time to failure for some types of components, and the literature seems to indicate increased use of this distribution in reliability models." (Banks and Carson, 1984:134)

The gamma and Weibull distributions both include the exponential distribution as a special case (Banks and Carson, 1984:132).

The exponential distribution is often used in life testing applications because it is easy to apply (Kapur and Lamberson, 1977:233). Also, in the case of a total system composed of many components with different failure distributions, the time to system failure distribution will approach the exponential (Kapur and Lamberson, 1977:236). Of particular interest in this thesis is the two-parameter exponential distribution. This distribution

form is used in nonzero minimum life situations, i.e., situations in which there is an initial period of no failures (Kapur and Lamberson, 1977:258).

Following is additional information concerning the four distributions of interest.

The Exponential Distribution. The two-parameter exponential distribution has a probability density function given by

$$f(x;m,d) = \begin{cases} (1/m)\exp\{-(x-d)/m\}, & x \geq d>0,\ m>0 \\ 0 & \text{, elsewhere} \end{cases} \tag{1}$$

where m equals the mean, and the parameter d is the minimum life. The mean of this probability density function is m+d (Kapur and Lamberson, 1977:258).

The cumulative distribution function is then given by

$$F(x) = \begin{cases} 0 & , x < d \\ \int_0^x (1/m)\exp\{-(t-d)/m\}\,dt & \\ \quad = 1 - \exp\{-(x-d)/m\} & , x \geq d>0,\ m>0 \end{cases} \tag{2}$$

In the situation where n items are placed on life test, and the test is terminated at the time of the rth failure, minimum variance, unbiased, maximum likelihood

estimators for m and d are m' and d' respectively and are defined by

$$m' = \frac{\sum_{i=2}^{r} (x_i - x_1) + (n-r)(x_r - x_1)}{(r-1)} \tag{3}$$

and

$$d' = x_1 - (m'/n) \tag{4}$$

where x_1 is the first ordered statistic (Kapur and Lamberson, 1977:258-259). This thesis primarily considered the case where n=r, i.e., uncensored samples.

<u>The Weibull Distribution</u>. The cumulative distribution function for the three-parameter Weibull distribution is given as

$$F(x;b,a,d) = \begin{cases} 0 & , x<d \\ 1-\exp\{-((x-d)/(b-d))^a\} & , x \geq d \end{cases} \tag{5}$$

where a>0, b>0, and d≥0. a is the shape parameter or the Weibull slope, b is the scale parameter or the characteristic life, and d is the location parameter or the minimum life (Kapur and Lamberson, 1977:292).

The Gamma Distribution. The probability density function for the gamma distribution is

$$f(x;a,b) = \begin{cases} \{1/(b^a \Gamma(a))\}(x^{a-1})\exp\{-(x-d)/b\} & \text{for } x>0,\ a>0,\ b>0 \\ 0 & \text{, elsewhere} \end{cases} \quad (6)$$

where a is the shape parameter, b is the scale parameter, and $\Gamma(a)$ is a value of the gamma function defined by

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx \quad (7)$$

Through integration by parts, the above equation reduces to

$$\Gamma(a) = (a-1)\Gamma(a-1) \quad (8)$$

for a>0, and

$$\Gamma(a) = \Gamma(a-1)! \quad (9)$$

when a is a positive integer. (Miller and Freund, 1977:117) The exponential distribution is a special case of the gamma distribution when a is equal to one.

The cumulative distribution function is given by

$$F(t) = \begin{cases} 0 & , t<0 \\ \int_0^x \{1/(b^a \Gamma(a))\} \; t^{a-1} \exp\{-(t-d)/b\} dt, & 0 \leq t \leq x \end{cases} \qquad (10)$$

If a is an integer, successive integration by parts yields

$$F(x) = \frac{\sum_{k=a}^{\infty} \{(1/b)x\}^k \exp\{-(x-d)/b\}}{k!} \qquad (11)$$

(Kapur and Lamberson, 1977:25).

The Log-normal Distribution. "The log-normal distribution occurs in practice whenever we encounter a random variable which is such that its logarithm has a normal distribution." (Miller and Freund, 1977:114)

The probability density function for the log-normal distribution is given by

$$f(t) = \begin{cases} \{1/(\sigma t\sqrt{2\pi})\} \\ \quad \exp\{-(1/2)((\ln t - \mu)/\sigma)^2\}, & t \geq 0 \\ 0 & , t<0 \end{cases} \qquad (12)$$

where $-\infty<\mu<\infty$ and $\sigma>0$. Here, the logarithm of the random variable t has a normal distribution, i.e., a random variable x defined as x=ln t will be normally distributed with a mean of μ and a standard deviation of σ (Kapur and Lamberson, 1977:19). The mean of the log-normal distr bution is

$$E(t) = \exp\{\mu + \sigma^2/2\} \tag{13}$$

and the variance is

$$V(t) = \{e^{2\mu+\sigma^2}\}\ \{e^{\sigma^2} - 1\} \tag{14}$$

(*Kapur and Lamberson*, 1977:20).

The log-normal distribution has a cumulative distributon function of

$$F(t) = \begin{cases} 0 & , \tau<0 \\ \int_0^t (1/(\tau\sigma\sqrt{2\pi})) \exp\{-(1/2)((\ln \tau - \mu)/\sigma^2)\}d\tau, & 0\leq\tau\leq t \end{cases} \tag{15}$$

(Kapur and Lamberson, 1977:20).

Robust Estimation

The parameters of a suspected underlying distribution can be estimated using the information provided by a random sample from that population. Statistical methods that produce estimates that are relatively insensitive to assumptions about the underlying distribution have been termed robust methods (Crow and Siddiqui, 1967). These robust parameter estimates should continue to perform well under moderate deviations from the suspected distribution (Parr and Schucany, 1979:2).

In this thesis the two-parameter negative exponential distribution was used as a baseline, with the Weibull, gamma, and log-normal distributions providing the "moderate deviations". These distributions were chosen because of their similarity to each other (see Appendix A). Their probability density functions all contain an exponential factor, and, for small shape parameters, they have similar curves when graphed. With small sample sizes, like those used in this thesis, it may be possible to fit the data with all four distributions, and impossible to say with certainty which distribution is the actual underlying distribution.

Minimum Distance Estimation

Minimum distance estimation has been considered as a method for deriving robust estimators (Parr and Schucany, 1979 and 1982). The "distance" referred to here is a discrepancy measure between an empirical distribution function and a theoretical cumulative distribution function (Parr and Schucany, 1979:3).

The theoretical cumulative distribution function used in this thesis is given in equation (2). The mean, m, will be estimated using equation (3), and the minimum life, d, will be initially estimated using equation (4). The F(x) values so computed are the z_i values used in the distance statistics. The initial value for the minimum life is then varied through several iterations of the distance statistics until the distance is minimized. The value of the minimum life that achieves this minimum distance is recorded as an estimate for the actual minimum life.

Three distance statistics were tried in this thesis: the Kolmogorov, Cramer-von Mises, and Anderson-Darling. The computing formulas for these statistics are given below; their theoretical development is beyond the scope of this thesis.

The Kolmogorov Statistics.

$$D^+ = \max_{1 \le i \le n} ((i/n) - z_i) \tag{16}$$

$$D^- = \max_{1 \le i \le n} (z_i - (i-1)/n) \tag{17}$$

$$D = \max\ (D^+,\ D^-) \tag{18}$$

The Cramer-von Mises Statistic.

$$W^2 = \sum_{i=1}^{n} (z_i - (2i-1)/2n)^2 + (1/12n) \tag{19}$$

The Anderson-Darling Statistic.

$$A^2 = -\{\sum_{i=1}^{n} (2i-1)(\ln z_i + \ln(1-z_{n+1-i}))\}/n - n \tag{20}$$

(Stephens, 1974:730-731).

Measures of Effectiveness

Mean Square Error. The mean square error was computed using the following formula:

$$MSE = \frac{\sum_{i=1}^{s} (d - d')^2}{s} \tag{21}$$

where d was the actual minimum life of the underlying distributions the samples were generated from, d' was the estimated minimum life, and s was the number of samples generated. The sample sizes were varied (8, 12, 16, and 20), and the number of samples taken for each configuration of input parameters and sample size remained constant at one thousand.

A small mean square error would indicate that the estimated minimum life deviated little from the actual minimum life, while a large mean square error would indicate that it deviated to a greater extent.

Relative Efficiency. Relative efficiency was computed as follows:

$$\text{Relative Efficiency} = \frac{\text{MSE of "best" estimator}}{\text{MSE of comparison estimator}} \qquad (22)$$

For the underlying exponential distribution, the best estimator for the minimum life was chosen to be d' (see equation (4)). For the other three distributions, the first ordered statistic (x_1) was chosen as the best estimator, primarily because it was easy to obtain. A relative efficiency greater than one indicated the comparison estimator performed better than the "best" estimator. Five different minimum life estimates were compared in this thesis: the exponential estimate (d'; see equation (4)), the first ordered statistic, and the three minimum distance estimates.

III. Methodology

Overview of Method

The two-parameter negative exponential distribution was chosen as the baseline for computing the minimum life estimates. Random samples were taken from the exponential, Weibull, gamma, and log-normal distributions. Sample sizes of 8, 12, 16, and 20 were used. Each sample was assumed to be from an exponential distribution, and the mean and location parameters were estimated using equations (3) and (4). These estimated parameters were then used in the theoretical cumulative distribution function used in computing the Kolmogorov, Cramer-von Mises, and Anderson-Darling distances (see equations (2), (18), (19), and (20)).

One thousand runs were made for each configuration of sample size and input parameters. The mean square errors and relative efficiencies for each of the minimum life estimates were then computed.

Random Number Generation

The Monte Carlo method is a technique that uses random or pseudorandom numbers for solution of a model. There are arithmetic codes available at many computer centers for generating sequences of pseudorandom digits, where each digit (0 through 9) occurs with approximately

equal likelihood (Rubinstein, 1981:11).

The pseudorandom number generators used in this thesis were routines from the International Mathematics and Statistical Libraries, Inc. (IMSL). Four routines were used: GGEXN to produce exponential random deviates, GGWIB to produce Weibull random deviates, GGAMR to produce gamma random deviates, and GGNLG to produce log-normal random deviates. Ten was added to each of the random deviates to simulate a minimum life of ten. This value was chosen arbitrarily.

Parameter Estimation

Maximum Likelihood Estimation. For the nonzero minimum life situation the mean and minimum life were estimated using equations (3) and (4) respectively. The underlying distribution was assumed to be exponential.

Distance Estimation. The theoretical cumulative distribution function was assumed to be exponential, and the maximum likelihood estimates for the mean and minimum life were used in equation (2) in the computation of the z_i values used in the distance estimation equations (equations (16), (17), (19), and (20)).

Investigation of Robustness of Estimator

One thousand samples from each configuration of input parameters and sample size were taken, and five

estimates for the minimum life were made. The mean, standard deviation, mean square error, and relative efficiency were then computed. The results are presented in table form (see Appendices B and C) and are analyzed in Chapter 4.

IV. Results

Appendix A contains Figures 1 through 4. These are representative graphs of the probability density functions of the four families of distributions considered in this thesis.

Appendices B and C contain tables summarizing the mean square errors and relative efficiencies of the minimum life estimators. There were five estimators considered: the exponential estimator (d'; see equation (4)), the first ordered statistic, and the three minimum distance estimators (Kolmogorov, Cramer-von Mises, and Anderson-Darling).

An inspection of Tables I through VIII shows that in most cases the exponential estimator (d') was superior, with the Anderson-Darling estimator being next best. However, the minimum distance estimators require much more time computationally than the exponential estimator. In order for one of them to be considered for robust estimation purposes it would have to perform much better than the exponential estimator.

The first ordered statistic was superior when the shape parameter equaled .5. More runs with shape parameters less than one would be needed to determine just when the first ordered statistic becomes a better estimator than the exponential estimator.

From a visual inspection of the graphs in Appendix A it may be argued that as shape parameters become greater than one, the underlying distribution may no longer be a "moderate" deviation from the exponential distribution. In these cases, a robust estimator based on the exponential distribution could not be expected to produce good results. Many more runs with different shape parameter values would be needed to better define what could be considered a moderate deviation from the exponential distribution.

The IMSL routine ZXMIN was used to minimize the minimum distance functions. It was noted that the Kolmogorov function was often terminated due to rounding errors before it reached a minimum value. The Kolmogorov values were left in the tables for comparison purposes should someone develop an algorithm that would truely minimize the function.

V. Conclusions and Recommendations

The exponential estimator for the minimum life (d') may provide a robust estimate for the minimum life of distributions that deviate moderately from the exponential. What constitutes a moderate deviation still needs to be determined. Variations in the shape parameter greatly affected the mean square errors of the estimators. More tests would have to be made in order to determine the boundaries of acceptable shape parameter values for this particular robust estimation method.

The use of histograms might help with determining if the shape parameter is too large to use the exponential estimator. However, the small sample sizes typically used with robust estimation may not provide enough data.

The minimum distance estimators did not do well with the asymmetric distributions used in this thesis. Further study using a modified minimum distance method might be feasible, but tremendous improvement in the mean square errors would have to be achieved in order to justify the much greater expense for the computer time required.

APPENDIX A

GRAPHS OF REPRESENTATIVE PROBABILITY DENSITY FUNCTIONS

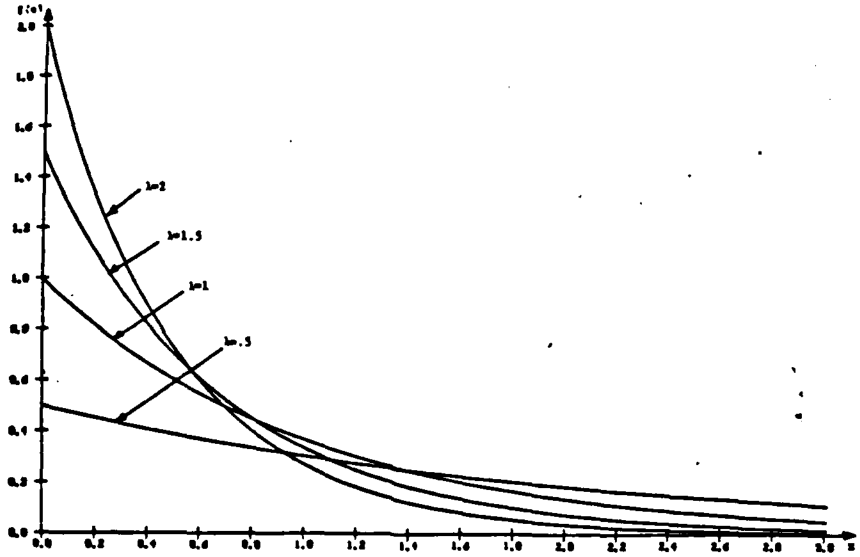


Figure 4.10. PDFs for several exponential distributions.

Figure 1. PDFs for several exponential distributions. (Banks and Carson, 1984:143)

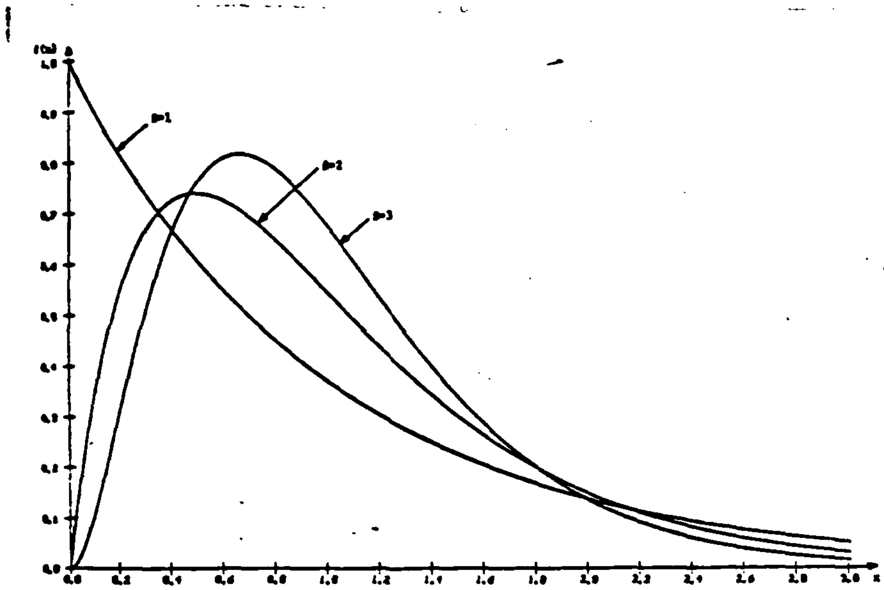


Figure 4.11. PDFs for several gamma distributions **when $\theta = 1$.**

Figure 2. PDFs for several gamma distributions when θ=1. (Banks and Carson, 1984:145)

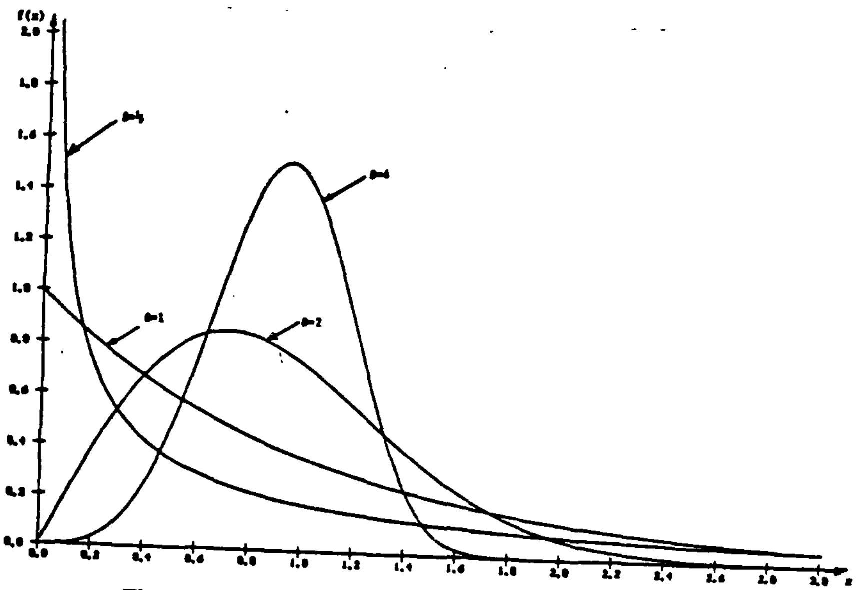


Figure 4.20. Weibull PDFs for $\nu = 0$, $\alpha = 1$, $\beta = \frac{1}{2}, 1, 2, 4$.

Figure 3. Weibull PDFs for $\nu=0$, $\alpha=1$, $\beta=\frac{1}{2}$, 1, 2, 4. (Banks and Carson, 1984:156)

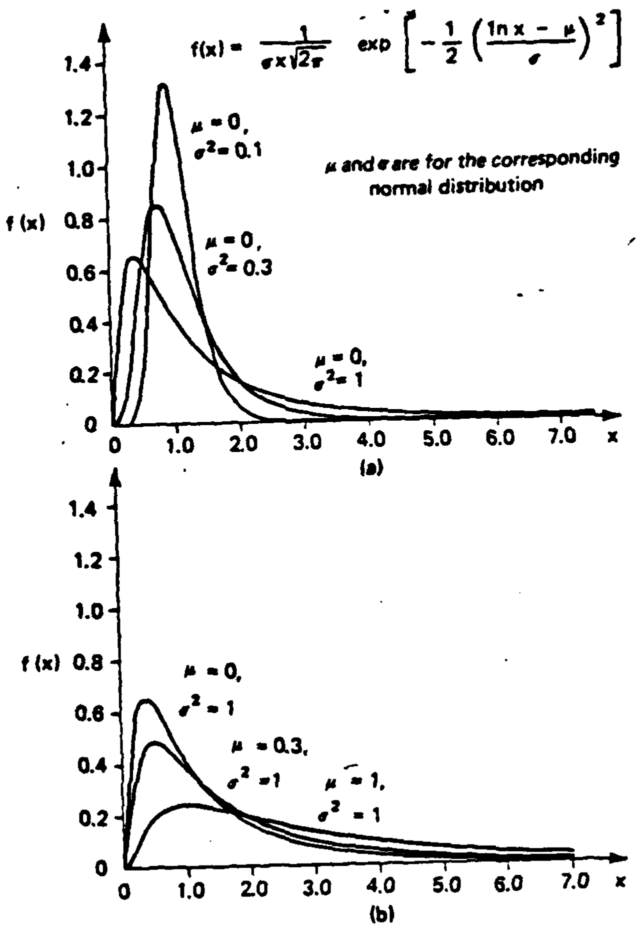


Figure 2-12 Lognormal density function and illustrations.

Figure 4. Lognormal density function and illustrations. (Pritsker, 1984:35)

APPENDIX B

TABLES I - IV

MEAN SQUARE ERRORS

FOR LOCATION PARAMETER ESTIMATES

TABLE I

MEAN SQUARE ERRORS FOR LOCATION PARAMETER ESTIMATES

Sample size = 8 Number of samples = 1000 Scale = 1.0 Actual location = 10.0

Underlying Distribution	Exponential	Kolmogorov	Cramer-von Mises	Anderson-Darling	First Ordered Statistic
Exponential	.01903493	.03040718	.02456631	.02166006	.03096700
Gamma (Shape=.5)	.00496621	.01268730	.01280247	.01993863	.00196061
Gamma (Shape=1)	.01636018	.02781988	.02193112	.01938951	.03107041
Gamma (Shape=2)	.20808277	.33917229	.29194259	.26831421	.37270136
Gamma (Shape=4)	2.27024362	3.16449708	2.86793117	2.70749254	3.20574208
Weibull (Shape=.5)	.10848376	.66178930	.72816813	.64841812	.00594447
Weibull (Shape=2)	.08357090	.12666993	.11032235	.10246450	.12259574
Weibull (Shape=4)	.26394937	.33179181	.30651963	.29430514	.31294842
Log-normal (Mu=0, Sigma=1)	.04933586	.11474873	.10146697	.08847986	.11544122
Log-normal (Mu=1, Sigma=1)	.36454543	.84793569	.74974529	.65378321	.85300166
Log-normal (Mu=1, Sigma=2)	46.34955769	683.06566996	725.50554469	694.40778218	.27588023

TABLE 11

MEAN SQUARE ERRORS FOR LOCATION PARAMETER ESTIMATES

Sample size = 12 Number of samples = 1000 Scale = 1.0 Actual location = 10.0

Underlying Distribution	Exponential	Kolmogorov	Cramer-von Mises	Anderson-Darling	First Ordered Statistic
Exponential	.00825412	.01612343	.01211386	.01028829	.01448355
Gamma (Shape=.5)	.00187507	.00912040	.00960571	.00780204	.00021618
Gamma (Shape=1)	.00831749	.01607652	.01172204	.01001081	.01533926
Gamma (Shape=2)	.13125865	.24755482	.19072652	.17231801	.22157253
Gamma (Shape=4)	1.77544787	2.68564423	2.24885111	2.12038707	2.36568072
Weibull (Shape=.5)	.04269490	.49946320	.59139745	.52212962	.00140613
Weibull (Shape=2)	.05911997	.10117417	.07900767	.07335833	.08292076
Weibull (Shape=4)	.22761376	.29831629	.25932683	.25177314	.26060644
Log-normal (Mu=0, Sigma=1)	.02679330	.06608218	.05594435	.04847688	.06582990
Log-normal (Mu=1, Sigma=1)	.19797721	.48832995	.41337676	.35819966	.48642083
Log-normal (Mu=1, Sigma=2)	10.85425133	379.88901769	414.50040398	401.46647445	.07101580

TABLE III

MEAN SQUARE ERRORS FOR LOCATION PARAMETER ESTIMATES

Sample size = 16   Number of samples = 1000   Scale = 1.0   Actual location = 10.0

Underlying Distribution	Exponential	Kolmogorov	Cramer-von Mises	Anderson-Darling	First Ordered Statistic
Exponential	.00400274	.00919027	.00622299	.00512922	.00766615
Gamma (Shape=.5)	.00092574	.00700240	.00813741	.00654649	.00009832
Gamma (Shape=1)	.00417857	.00997089	.00695218	.00567903	.00807056
Gamma (Shape=2)	.11543896	.22155558	.16249418	.14935486	.18138964
Gamma (Shape=4)	1.59144799	2.48393938	1.97038117	1.87767753	2.01356827
Weibull (Shape=.5)	.01949433	.34535504	.42827888	.37191187	.00034133
Weibull (Shape=2)	.04527748	.08582005	.06049030	.05661885	.06168353
Weibull (Shape=4)	.19979636	.27195232	.22441120	.21978098	.22462187
Log-normal (Mu=0, Sigma=1)	.02107707	.04436279	.03827787	.03281292	.04816671
Log-normal (Mu=1, Sigma=1)	.15573966	.32788384	.28283742	.24245387	.35590655
Log-normal (Mu=1, Sigma=2)	6.82903355	494.14936214	542.71922742	532.43534202	.03857610

TABLE IV

MEAN SQUARE ERRORS FOR LOCATION PARAMETER ESTIMATES

Sample size = 20   Number of samples = 1000   Scale = 1.0   Actual location = 10.0

Underlying Distribution	Exponential	Kolmogorov	Cramer-von Mises	Anderson-Darling	First Ordered Statistic
Exponential	.00211702	.00681984	.00429123	.00326237	.00432054
Gamma (Shape=.5)	.00063100	.00689781	.00821403	.00655305	.00005761
Gamma (Shape=1)	.00263234	.00746289	.00500266	.00397317	.00498450
Gamma (Shape=2)	.08444084	.17465836	.11937101	.11030557	.13022007
Gamma (Shape=4)	1.35872495	2.23061526	1.65964137	1.59664637	1.67958609
Weibull (Shape=.5)	.01308104	.34559249	.45693845	.40027623	.00008449
Weibull (Shape=2)	.03591466	.07393746	.04762530	.04501656	.04807682
Weibull (Shape=4)	.17878317	.25116673	.19831651	.19541636	.19840608
Log-normal (Mu=0, Sigma=1)	.01862048	.03706537	.03165501	.02839316	.03877545
Log-normal (Mu=1, Sigma=1)	.13758778	.27392223	.23390062	.20979521	.28651398
Log-normal (Mu=1, Sigma=2)	3.59798898	346.07151915	403.74906961	394.07538091	.02796067

APPENDIX C

TABLES V - VIII

RELATIVE EFFICIENCIES

OF LOCATION PARAMETER ESTIMATES

TABLE V

RELATIVE EFFICIENCIES OF LOCATION PARAMETER ESTIMATES

Sample size = 8 Number of samples = 1000 Scale = 1.0 Actual location = 10.0

Underlying Distribution	Exponential	Kolmogorov	Cramer-von Mises	Anderson-Darling	First Ordered Statistic
Exponential	1.00000000	.62600127	.77483902	.87880324	.61468443
Gamma (Shape=.5)	.39478874	.15453288	.15314279	.09833198	1.00000000
Gamma (Shape=1)	1.89914846	1.11684200	1.41672692	1.60243400	1.00000000
Gamma (Shape=2)	1.79112070	1.09885557	1.27662552	1.38904816	1.00000000
Gamma (Shape=4)	1.41206964	1.01303367	1.11778906	1.18402619	1.00000000
Weibull (Shape=.5)	.05479593	.00898242	.00816359	.00916765	1.00000000
Weibull (Shape=2)	1.46696679	.96783614	1.11125019	1.19647037	1.00000000
Weibull (Shape=4)	1.18563805	.94320720	1.02097351	1.06334676	1.00000000
Log-normal (Mu=0, Sigma=1)	2.33990494	1.00603483	1.13772217	1.30471743	1.00000000
Log-normal (Mu=1, Sigma=1)	2.33990494	1.00597448	1.13772194	1.30471637	1.00000000
Log-normal (Mu=1, Sigma=2)	.00595217	.00040389	.00038026	.00039729	1.00000000

TABLE VI

RELATIVE EFFICIENCIES OF LOCATION PARAMETER ESTIMATES

Sample size = 12   Number of samples = 1000   Scale = 1.0   Actual location = 10.0

Underlying Distribution	Exponential	Kolmogorov	Cramer-von Mises	Anderson-Darling	First Ordered Statistic
Exponential	1.00000000	.51193323	.68137787	.80228318	.56989627
Gamma (Shape=.5)	.11528973	.02370253	.02250499	.02770770	1.00000000
Gamma (Shape=1)	1.84421725	.95414003	1.30858262	1.53226908	1.00000000
Gamma (Shape=2)	1.68806039	.89504429	1.16172902	1.28583499	1.00000000
Gamma (Shape=4)	1.33244166	.88086154	1.05195080	1.11568343	1.00000000
Weibull (Shape=.5)	.03293445	.00281529	.00237764	.00269307	1.00000000
Weibull (Shape=2)	1.40258467	.81958436	1.04952806	1.13035237	1.00000000
Weibull (Shape=4)	1.14495026	.87359104	1.00493435	1.03508435	1.00000000
Log-normal (Mu=0, Sigma=1)	2.45695365	.99618234	1.17670332	1.35796495	1.00000000
Log-normal (Mu=1, Sigma=1)	2.45695365	.99609050	1.17670095	1.35796005	1.00000000
Log-normal (Mu=1, Sigma=2)	.00654267	.00018694	.00017133	.00017689	1.00000000

TABLE VII

RELATIVE EFFICIENCIES OF LOCATION PARAMETER ESTIMATES

Sample size = 16 Number of samples = 1000 Scale = 1.0 Actual location = 10.0

Underlying Distribution	Exponential	Kolmogorov	Cramer-von Mises	Anderson-Darling	First Ordered Statistic
Exponential	1.00000000	.43554114	.64321872	.78037944	.52213165
Gamma (Shape=.5)	.10620633	.01404087	.01208244	.01501870	1.00000000
Gamma (Shape=1)	1.93141396	.80941170	1.16086774	1.42111550	1.00000000
Gamma (Shape=2)	1.57130346	.81870942	1.11628390	1.21448768	1.00000000
Gamma (Shape=4)	1.26524290	.81063503	1.02191814	1.07237171	1.00000000
Weibull (Shape=.5)	.01750908	.00098834	.00079698	.00091777	1.00000000
Weibull (Shape=2)	1.36234444	.71875425	1.01972595	1.08945209	1.00000000
Weibull (Shape=4)	1.12425412	.82596050	1.00093881	1.02202601	1.00000000
Log-normal (Mu=0, Sigma=1)	2.28526597	1.08574578	1.25834373	1.46791928	1.00000000
Log-normal (Mu=1, Sigma=1)	2.28526597	1.08546537	1.25834322	1.46793511	1.00000000
Log-normal (Mu=1, Sigma=2)	.00564884	.00007807	.00007108	.00007245	1.00000000

TABLE VIII

RELATIVE EFFICIENCIES OF LOCATION PARAMETER ESTIMATES

Sample size = 20 Number of samples = 1000 Scale = 1.0 Actual location = 10.0

Underlying Distribution	Exponential	Kolmogorov	Cramer-von Mises	Anderson-Darling	First Ordered Statistic
Exponential	1.00000000	.31042024	.49333519	.64891913	.48998802
Gamma (Shape=.5)	.09130287	.00835228	.00701391	.00879169	1.00000000
Gamma (Shape=1)	1.89356563	.66790463	.99637050	1.25454131	1.00000000
Gamma (Shape=2)	1.54214556	.74557021	1.09088527	1.18053947	1.00000000
Gamma (Shape=4)	1.23614871	.75296987	1.01201749	1.05194620	1.00000000
Weibull (Shape=.5)	.00645921	.00024449	.00018491	.00021109	1.00000000
Weibull (Shape=2)	1.33864058	.65023631	1.00948080	1.06798073	1.00000000
Weibull (Shape=4)	1.10975816	.78993774	1.00045164	1.01529926	1.00000000
Log-normal (Mu=0, Sigma=1)	2.08240873	1.04613696	1.22493901	1.36566182	1.00000000
Log-normal (Mu=1, Sigma=1)	2.08240873	1.04596834	1.22493895	1.36568409	1.00000000
Log-normal (Mu=1, Sigma=2)	.00777119	.00008079	.00006925	.00007095	1.00000000

APPENDIX D

SAMPLE COMPUTER PROGRAM

```
      PROGRAM GKWA2(GKWA20,TAPE10=GKWA20)
C
C     UNDERLYING DISTRIBUTION IS GAMMA.
C     UNCENSORED SAMPLES.
C     FOUR SAMPLE SIZES - 8,12,16, AND 20.
C     SCALE=1.0
C     SHAPE=2.0 AND 4.0
C
C
C     PROGRAM FUNCTION:
C
C        - GENERATES GAMMA RANDOM NUMBERS.
C        - SORTS RANDOM NUMBERS.
C        - SAVES THE FIRST ORDERED STATISTIC.
C        - ESTIMATES SCALE AND LOCATION PARAMETERS
C             ASSUMING EXPONENTIAL DISTRIBUTION.
C        - REESTIMATES LOCATION PARAMETER USING
C             KOLMORGOROV DISTANCE.
C        - REESTIMATES LOCATION PARAMETER USING
C             CRAMER-VON MISES DISTANCE.
C        - REESTIMATES LOCATION PARAMETER USING
C             ANDERSON-DARLING DISTANCE.
C        - COMPUTES MEANS, STANDARD DEVIATIONS, MEAN
C             SQUARE ERRORS, AND RELATIVE EFFICIENCIES.
C
C
C     VARIABLES USED:
C
C        N - SAMPLE SIZE.
C        I,L,P,Q - LOOP COUNTERS AND ARRAY INDICES.
C        NRUNS - NUMBER OF RUNS (NUMBER OF SAMPLES).
C        IOPT - ZXMIN INPUT - OPTIONS SELECTION.
C        MAXFN - ZXMIN INPUT - MAXIMUM NUMBER OF FUNCTION
C             EVALUATIONS ALLOWED.
C        NSIG - ZXMIN INPUT - NUMBER OF DIGITS ACCURACY
C             REQUIRED.
C        NPARAM - ZXMIN INPUT - NUMBER OF PARAMETERS.
C        IER - ZXMIN OUTPUT - ERROR PARAMETER.
C        D - ACTUAL MINIMUM LIFE (LOCATION PARAMETER).
C        M - SCALE PARAMETER.
C        TEMP - TEMPORARY VARIABLE.
C        R(N) - RANDOM NUMBER ARRAY.
C        Z(N) - Z VALUE ARRAY (CDF).
C        X(NPARAM) - ZXMIN INPUT/OUTPUT - PARAMETER
C             VALUE.
C        X1SAVE - PREVIOUS X(NPARAM).
C        F - ZXMIN OUTPUT - VALUE OF FUNCTION BEING
C             MINIMIZED.
C        FSAVE - PREVIOUS F.
C        H(1),G(1) - ZXMIN OUTPUT - WORKING VECTORS.
C        W(3) - ZXMIN OUTPUT - ZXMIN OUTPUT -
C             VECTOR.
C             W(1) - NORM OF THE GRADIENT.
C             W(2) - NUMBER OF FUNCTION EVALUATIONS
C                  PERFORMED.
C             W(3) - AN ESTIMATE OF THE NUMBER OF
C                  SIGNIFICANT DIGITS IN THE FINAL
C                  PARAMETER ESTIMATE.
C        SHAPE - SHAPE PARAMETER (USED IN GAMMA AND
C             WEIBULL DISTRIBUTIONS ONLY).
C        WK(40) - WORKING VECTOR (USED IN GAMMA
C             DISTRIBUTION ONLY).
C        MU - MEAN (USED IN LOG-NORMAL DISTRIBUTION
```

```
C            ONLY).
C        SIGMA - STANDARD DEVIATION (USED IN LOG-NORMAL
C             DISTRIBUTION ONLY).
C        ED(NRUNS) - INITIAL ESTIMATE OF THE LOCATION
C             PARAMETER, ASSUMING AN EXPONENTIAL
C             DISTRIBUTION.
C        EM(NRUNS) - ESTIMATE OF THE SCALE PARAMETER,
C             ASSUMING AN EXPONENTIAL DISTRIBUTION.
C        X1(NRUNS) - FIRST ORDERED STATISTIC.
C        EDK(NRUNS) - KOLMOGOROV LOCATION PARAMETER
C             ESTIMATE.
C        EDW(NRUNS) - CRAMER-VON MISES LOCATION
C             PARAMETER ESTIMATE.
C        EDA(NRUNS) - ANDERSON-DARLING LOCATION
C             PARAMETER ESTIMATE.
C        DPLUS,DMINUS - KOLMOGOROV MAXIMUM POSITIVE
C             AND NEGATIVE DEVIATIONS.
C        DSUM,KSUM,WSUM,ASUM,MSUM,XSUM - INTERMEDIATE
C             VALUES.
C        XD,XK,XW,XA,XM,XX - SAMPLE MEANS.
C        SD,SK,SW,SA,SM,SX - SAMPLE STANDARD
C             DEVIATIONS.
C        MSED,MSEK,MSEW,MSEA,MSEM,MSEX - MEAN SQUARE
C             ERRORS.
C        DSEED - INPUT SEED TO RANDOM NUMBER
C             GENERATORS.
C
C
C      SUBROUTINES USED:
C
C        SORT - BUBBLE SORTS RANDOM NUMBERS, PLACING
C               SMALLEST NUMBER IN R(1).
C
C        GGEXN - (PROGRAM EKWA ONLY) IMSL ROUTINE TO
C               GENERATE EXPONENTIAL RANDOM NUMBERS.
C
C        GGWIB - (PROGRAMS WKWA1 AND WKWA2 ONLY) IMSL
C               ROUTINE TO GENERATE WEIBULL RANDOM
C               NUMBERS.
C
C        GGAMR - (PROGRAMS GKWA1 AND GKWA2 ONLY) IMSL
C               ROUTINE TO GENERATE GAMMA RANDOM
C               NUMBERS.
C
C        GGNLG - (PROGRAMS LKWA1 AND LKWA2 ONLY) IMSL
C               ROUTINE TO GENERATE LOG-NORMAL
C               RANDOM NUMBERS.
C
C
C      INPUT FILES:  NONE
C
C
C      OUTPUT FILES:
C
C        EKWAO - PROGRAM EKWA
C        GKWA1O - PROGRAM GKWA1
C        GKWA2O - PROGRAM GKWA2
C        WKWA1O - PROGRAM WKWA1
C        WKWA2O - PROGRAM WKWA2
C        LKWA1O - PROGRAM LKWA1
C        LKWA2O - PROGRAM LKWA2
C
C
C**********************************************************
C
      COMMON R,N,P,ED,EM,X1,EDK,EDW,EDA,XISAVE,FSAVE
```

```
      EXTERNAL KDIST,WDIST,ADIST
      INTEGER N,I,L,P,Q,NRUNS,
     +        IOPT,MAXFN,NSIG,NPARAM,IER
      REAL D,M,TEMP,R(20),Z(20),X1SAVE,FSAVE,
     +     X(1),H(1),G(1),F,W(3),
     +     ED(1000),EM(1000),X1(1000),
     +     EDK(1000),EDW(1000),EDA(1000),
     +     DPLUS,DMINUS,SHAPE,MU,SIGMA,WK(40),
     +     DSUM,KSUM,WSUM,ASUM,MSUM,XSUM,
     +     XD,XK,XW,XA,XM,XX,
     +     SD,SK,SW,SA,SM,SX,
     +     MSED,MSEK,MSEW,MSEA,MSEM,MSEX
      DOUBLE PRECISION DSEED
C     ** UERSET SUPPRESSES TERMINAL ERRORS.
      CALL UERSET(0,LEVOLD)
      DSEED=123457.D0
      NRUNS=1000
      M=1.0
      D=10.0
      NPARAM=1
      NSIG=4
      MAXFN=500
      IOPT=0
C
C
C     ** TWO SHAPES (2.0 AND 4.0)
C
C
      SHAPE=2.0
      DO 8 L=1,2
C
C
C     ** RUN 4 SAMPLE SIZES
C
C
      N=8
      DO 9 Q=1,4
C
C
C     ** MAKE 1000 RUNS
C
C
      DO 10 P=1,NRUNS
C
C
C     ** GENERATE GAMMA RANDOM NUMBERS
C
C
        CALL GGAMR(DSEED,SHAPE,N,WK,R)
C
C
C     ** ADD OFFSET FOR MINIMUM LIFE
C
C
        DO 6 I=1,N
          R(I)=R(I)+D
    6   CONTINUE
C
C
C     ** SORT RANDOM NUMBERS
C
C
        CALL SORT(N,R)
C
C
C     ** SAVE FIRST ORDERED STATISTIC
```

```
C
C
         X1(P)=R(1)
C
C
C     ** ESTIMATE SCALE AND LOCATION PARAMETERS
C
C
         XSUM=0.0
        DO 50 I=2,N
           XSUM=XSUM+R(I)-R(1)
   50   CONTINUE
         EM(P)=XSUM/FLOAT(N-1)
         ED(P)=R(1)-EM(P)/FLOAT(N)
C
C
C     ** REESTIMATE MINIMUM LIFE USING KOLMOGOROV
C         DISTANCE.
C
C
         X(1)=ED(P)
         CALL ZXMIN(KDIST,NPARAM,NSIG,MAXFN,IOPT,
     +              X,H,G,F,W,IER)
         EDK(P)=X(1)
C
C
C     ** REESTIMATE MINIMUM LIFE USING CRAMER-VON
C         MISES DISTANCE.
C
C
         X(1)=ED(P)
         X1SAVE=R(1)
         CALL ZXMIN(WDIST,NPARAM,NSIG,MAXFN,IOPT,
     +              X,H,G,F,W,IER)
         EDW(P)=X(1)
C
C
C     ** REESTIMATE MINIMUM LIFE USING
C         ANDERSON-DARLING DISTANCE.
C
C
         X(1)=ED(P)
         X1SAVE=R(1)
         FSAVE=.999
         CALL ZXMIN(ADIST,NPARAM,NSIG,MAXFN,IOPT,
     +              X,H,G,F,W,IER)
         EDA(P)=X(1)
   10 CONTINUE
C
C
C     ** COMPUTE SAMPLE MEAN
C
C
      DSUM=0.0
      KSUM=0.0
      MSUM=0.0
      XSUM=0.0
      WSUM=0.0
      ASUM=0.0
      DO 98 I=1,NRUNS
         DSUM=DSUM+ED(I)
         KSUM=KSUM+EDK(I)
         MSUM=MSUM+EM(I)
         XSUM=XSUM+X1(I)
         WSUM=WSUM+EDW(I)
         ASUM=ASUM+EDA(I)
```

```
 85 CONTINUE
    XD=DSUM/FLOAT(NRUNS)
    XK=KSUM/FLOAT(NRUNS)
    XM=MSUM/FLOAT(NRUNS)
    XX=XSUM/FLOAT(NRUNS)
    XW=WSUM/FLOAT(NRUNS)
    XA=ASUM/FLOAT(NRUNS)
C
C
C   ** COMPUTE SAMPLE STANDARD DEVIATIONS
C
C
    DSUM=0.0
    KSUM=0.0
    MSUM=0.0
    XSUM=0.0
    WSUM=0.0
    ASUM=0.0
    DO 89 I=1,NRUNS
      DSUM=DSUM+(ED(I)-XD)**2.0
      KSUM=KSUM+(EDK(I)-XK)**2.0
      MSUM=MSUM+(EM(I)-XM)**2.0
      XSUM=XSUM+(X1(I)-XX)**2.0
      WSUM=WSUM+(EDW(I)-XW)**2.0
      ASUM=ASUM+(EDA(I)-XA)**2.0
 89 CONTINUE
    SD=(DSUM/FLOAT(NRUNS-1))**.5
    SK=(KSUM/FLOAT(NRUNS-1))**.5
    SM=(MSUM/FLOAT(NRUNS-1))**.5
    SX=(XSUM/FLOAT(NRUNS-1))**.5
    SW=(WSUM/FLOAT(NRUNS-1))**.5
    SA=(ASUM/FLOAT(NRUNS-1))**.5
C
C
C   ** COMPUTE MEAN SQUARE ERRORS
C
C
    DSUM=0.0
    KSUM=0.0
    MSUM=0.0
    XSUM=0.0
    WSUM=0.0
    ASUM=0.0
    DO 11 I=1,NRUNS
      DSUM=DSUM+(D-ED(I))**2.0
      KSUM=KSUM+(D-EDK(I))**2.0
      MSUM=MSUM+(M-EM(I))**2.0
      XSUM=XSUM+(D-X1(I))**2.0
      WSUM=WSUM+(D-EDW(I))**2.0
      ASUM=ASUM+(D-EDA(I))**2.0
 11 CONTINUE
    MSED=DSUM/FLOAT(NRUNS)
    MSEK=KSUM/FLOAT(NRUNS)
    MSEM=MSUM/FLOAT(NRUNS)
    MSEX=XSUM/FLOAT(NRUNS)
    MSEW=WSUM/FLOAT(NRUNS)
    MSEA=ASUM/FLOAT(NRUNS)
C
C
C   ** PRINT TABLE
C
C
    WRITE(10,800)
800 FORMAT('1','OUTPUT FROM GKWA2'//)
    WRITE(10,801)
801 FORMAT(1X,'UNDERLYING DISTRIBUTION IS GAMMA')
```

```
      WRITE(10,805)D,SCALE,SHAPE,N,NRUNS
  805 FORMAT(1X,'LOCATION (D) =',F10.3/
     +        1X,'SCALE (M) =',F10.3/
     +        1X,'SHAPE =',F10.3/
     +        1X,'SAMPLE SIZE (N) =',I3/
     +        1X,'NBR OF RUNS (NRUNS) =',I5///)
      WRITE(10,810)
  810 FORMAT(19X,'MEAN')
      WRITE(10,815)
  815 FORMAT(17X,'SQUARE',9X,'RELATIVE',26X,
     +        'STANDARD')
      WRITE(10,820)
  820 FORMAT(18X,'ERROR',7X,'EFFICIENCY',13X,
     +        'MEAN',8X,'DEVIATION')
      WRITE(10,825)MSED,MSEX/MSED,XD,SD
  825 FORMAT(3X,'EXP',4(5X,F12.8))
      WRITE(10,830)MSEK,MSEX/MSEK,XK,SK
  830 FORMAT(3X,'KOL',4(5X,F12.8))
      WRITE(10,835)MSEW,MSEX/MSEW,XW,SW
  835 FORMAT(3X,'CVM',4(5X,F12.8))
      WRITE(10,840)MSEA,MSEX/MSEA,XA,SA
  840 FORMAT(4X,'AD',4(5X,F12.8))
      WRITE(10,845)MSEX,XX,SX
  845 FORMAT(4X,'X1',5X,F12.8,17X,2(5X,F12.8))
      WRITE(10,850)MSEM,XM,SM
  850 FORMAT(1X,'SCALE',5X,F12.8,17X,2(5X,F12.8))
        N=N+4
    9 CONTINUE
      SHAPE=SHAPE*2.0
    8 CONTINUE
      STOP
      END
C     **************************************************
      SUBROUTINE SORT(N,R)
      INTEGER I,J,N
      REAL TEMP,R(N)
      DO 405 I=1,N-1
        DO 410 J=I+1,N
          IF(R(J) .LT. R(I)) THEN
C           ** SWAP
            TEMP=R(I)
            R(I)=R(J)
            R(J)=TEMP
          ENDIF
  410   CONTINUE
  405 CONTINUE
      RETURN
      END
C     *********************************************
      SUBROUTINE KDIST(NPARAM,X,F)
      COMMON R,N,P,ED,EM,X1,EDK,EDW,EDA,X1SAVE,FSAVE
      INTEGER NPARAM,N,P
      REAL X(NPARAM),DPLUS,DMINUS,F,TEMP,X1SAVE,FSAVE,
     +     Z(20),R(20),ED(1000),EM(1000),EDK(1000),
     +     X1(1000),EDW(1000),EDA(1000)
C     ** COMPUTE Z(I)
      DO 61 I=1,N
        Z(I)=1.0-EXP(-(R(I)-X(NPARAM))/EM(P))
   61 CONTINUE
C     ** FIND MAX D+
      DPLUS=0.0
      DO 70 I=1,N
        TEMP=FLOAT(I)/FLOAT(N)-Z(I)
        IF(TEMP .GT. DPLUS) DPLUS=TEMP
   70 CONTINUE
C     ** FIND MAX D-
```

```
      DMINUS=0.0
      DO 71 I=1,N
        TEMP=Z(I)-FLOAT(I-1)/FLOAT(N)
        IF(TEMP .GT. DMINUS) DMINUS=TEMP
   71 CONTINUE
      F=MAX(DPLUS,DMINUS)
      RETURN
      END
C     ********************************************
      SUBROUTINE WDIST(NPARAM,X,F)
      COMMON R,N,P,ED,EM,X1,EDK,EDW,EDA,X1SAVE,FSAVE
      INTEGER NPARAM,N,P
      REAL X(NPARAM),XSUM,F,TEMP,Z(20),R(20),X1SAVE,FSAVE,
     +     X1(1000),ED(1000),EM(1000),
     +     EDK(1000),EDW(1000),EDA(1000)
C
C     ** X1SAVE IS THE PREVIOUS VALUE OF X(NPARAM).
C        ZXMIN OCCASIONALLY PUTS AN EXTREMELY LARGE
C        NUMBER IN X(NPARAM).  WHEN THIS HAPPENS,
C        THE PREVIOUS VALUE IS USED.
C
      IF(X(NPARAM) .LE. R(1)) THEN
        X1SAVE=X(NPARAM)
      ELSE
        X(NPARAM)=X1SAVE
      ENDIF
C
C     ** COMPUTE Z(I)
C
      DO 62 I=1,N
        Z(I)=1.0-EXP(-(R(I)-X(NPARAM))/EM(P))
   62 CONTINUE
C
C     ** COMPUTE CRAMER-VON MISES DISTANCE
C
      XSUM=0.0
      DO 80 I=1,N
        TEMP=FLOAT(2*I-1)/FLOAT(2*N)
        XSUM=XSUM=XSUM+(Z(I)-TEMP)**2.0
   80 CONTINUE
      F=XSUM+FLOAT(1)/FLOAT(12*N)
      RETURN
      END
C     ********************************************
      SUBROUTINE ADIST(NPARAM,X,F)
      COMMON R,N,P,ED,EM,X1,EDK,EDW,EDA,X1SAVE,FSAVE
      INTEGER NPARAM,N,P,FLAG
      REAL X(NPARAM),F,Z(20),R(20),XSUM,TEMP,FSAVE,X1SAVE,
     +     X1(1000),ED(1000),EM(1000),
     +     EDK(1000),EDW(1000),EDA(1000)
C     ** MINIMUM LIFE NOT GREATER THAN FIRST
C        ORDERED STATISTIC.
      IF(X(NPARAM) .GT. R(1)) THEN
        X(NPARAM)=X1SAVE
      ELSE
        X1SAVE=X(NPARAM)
      ENDIF
C     ** COMPUTE Z(I)
      FLAG=0
      DO 63 I=1,N
        Z(I)=1.0-EXP(-(R(I)-X(NPARAM))/EM(P))
        IF( (Z(I) .LE. 0.) .OR. (Z(I) .GE. 1.) )FLAG=1
   63 CONTINUE
      IF (FLAG .EQ. 0) THEN
C       ** COMPUTE ANDERSON-DARLING DISTANCE
        XSUM=0.0
```

```
      DO 91 I=1,N
        TEMP=ALOG(Z(I))+ALOG(1.0-Z(N+1-I))
        XSUM=XSUM+FLOAT(2*I-1)*TEMP
 91   CONTINUE
      F=-XSUM/FLOAT(N)-FLOAT(N)
      FSAVE=F
    ELSE
      F=FSAVE
    ENDIF
    RETURN
    END
15.06.38.UCLP, CA, N1706H3,       0.640KLNS.
```

BIBLIOGRAPHY

Banks, Jerry and John S. Carson, II. Discrete-Event System Simulation. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1984.

Crow, E.L. and M. Siddiqui. "Robust Estimation of Location," Journal of the American Statistical Association, 62:353-388 (1967).

Kapur, K.C. and L.R. Lamberson. Reliability in Engineering Design. New York: John Wiley & Sons, 1977.

Miller, Irwin and John E. Freund. Probability and Statistics for Engineers (Second Edition). Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1977.

Parr, William C. and William R. Schucany. "Minimum Distance Estimation and Components of Goodness-of-fit Statistics," Journal of the Royal Statistical Society, 2:178-189 (1982).

----. Minimum Distance and Robust Estimation. Technical Report No. 136, Office of Naval Research Contract N00014-75-C-0439, Project NR 042-280. Dallas: Southern Methodist University, 5 October 1979.

Pritsker, A. Alan B. Introduction to Simulation and SLAM II (Second Edition). New York: John Wiley & Sons, 1984.

Rubinstein, Reuven Y. Simulation and the Monte Carlo Method. New York: John Wiley & Sons, 1981.

Stephens, M.A. "EDF Statistics for Goodness of Fit and Some Comparisons," Journal of the American Statistical Association, 69:730-737 (September 1974).

VITA

Captain Linda M. Allen was born on 11 May 1952 in Great Lakes, Illinois. She graduated from high school in Fulton, Missouri, in 1970, and enlisted in the USAF in 1971 after attending the University of Missouri for one year. In 1977 she was selected for the Airman Education and Commissioning Program and attended the University of Texas in San Antonio, from which she received a Bachelor of Science in Mathematics, Computer Science, and Systems Design in May 1979. Upon graduation, she received a commission in the USAF through OTS and was assigned to the 554th Range Group, Nellis AFB, Nevada, as a computer programmer. A year later she was reassigned to the 16th Surveillance Squadron, Shemya AB, Alaska, as a systems analyst for the COBRA DANE radar. Upon returning from this remote assignment, she was assigned to Headquarters Space Command, Peterson AFB, Colorado, as a computer staff officer, until entering the School of Engineering, Air Force Institute of Technology, in May 1984.

Permanent address: 3655 Cragwood Drive
Colorado Springs, CO 80907

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED

1b. RESTRICTIVE MARKINGS:

2a. SECURITY CLASSIFICATION AUTHORITY:

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE:

3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S): AFIT/GSO/MA/85D-1

5. MONITORING ORGANIZATION REPORT NUMBER(S):

6a. NAME OF PERFORMING ORGANIZATION: School of Engineering

6b. OFFICE SYMBOL (If applicable): AFIT/ENS

7a. NAME OF MONITORING ORGANIZATION:

6c. ADDRESS (City, State and ZIP Code): Air Force Institute of Technology, Wright-Patterson AFB, Ohio 45433

7b. ADDRESS (City, State and ZIP Code):

8a. NAME OF FUNDING/SPONSORING ORGANIZATION:

8b. OFFICE SYMBOL (If applicable):

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER:

8c. ADDRESS (City, State and ZIP Code):

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.

11. TITLE (Include Security Classification): See Box 19

12. PERSONAL AUTHOR(S): Linda M. Allen, B.S., Capt, USAF

13a. TYPE OF REPORT: MS Thesis

13b. TIME COVERED: FROM ________ TO ________

14. DATE OF REPORT (Yr., Mo., Day): 1985 December

15. PAGE COUNT: 56

16. SUPPLEMENTARY NOTATION:

17. COSATI CODES

FIELD	GROUP	SUB. GR.
12	01	

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number): Mathematics

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Title: ROBUST ESTIMATION OF THE LOCATION PARAMETER OF LIFE DISTRIBUTIONS

Thesis Advisor: Dr. Albert H. Moore
Professor of Mathematics

Approved for public release: IAW AFR 190-1.
16 JAN 86
LYNN E. WOLAVER
Dean for Research and Professional Development
Air Force Institute of Technology (ATC)
Wright-Patterson AFB OH 45433

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL: Dr. Albert H. Moore

22b. TELEPHONE NUMBER (Include Area Code): (513) 255-3098

22c. OFFICE SYMBOL: AFIT/ENC

DD FORM 1473, 83 APR   EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

thesis
This ~~investigation~~ determined that for time to failure distributions that are moderate deviations from the negative exponential distribution, a robust estimate of the minimum life could be arrived at by assuming the underlying distribution was exponential and using the minimum variance, unbiased, maximum likelihood estimator. It was found that estimators using the minimum distance statistics of Kolmogorov, Cramer-von Mises, and Anderson-Darling did not perform well with the asymmetric distributions explored in this thesis. However, they may still prove useful for life distributions with larger shape parameters (i.e., for distributions that are not "moderate" deviations from the negative exponential).

The analysis was accomplished by using Monte Carlo techniques to generate random samples of time to failure data from specific distributions, and using this empirical data to estimate the actual minimum life of the distribution. Five estimators were explored: the minimum variance, unbiased, maximum likelihood estimator of the two-parameter negative exponential distribution; the first ordered statistic; and the three minimum distance methods (the theoretical distribution was assumed negative exponential). The performance of these estimators was evaluated by comparing their mean square errors with the mean square error of the chosen "best" estimator.

END

FILMED

3-86

DTIC